Physics Letters B 692 (2010) 257–260

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Non-unitarity of the leptonic mixing matrix in the TeV-scale type-I seesaw model

Tommy Ohlsson, Christoph Popa, He Zhang *

Department of Theoretical Physics, School of Engineering Sciences, Royal Institute of Technology (KTH) – AlbaNova University Center, Roslagstullsbacken 21, 106 91 Stockholm, Sweden

ARTICLE INFO

Article history:
Received 7 July 2010
Accepted 29 July 2010
Available online 3 August 2010
Editor: T. Yanagida

Keywords:
Leptonic flavor mixing
Seesaw mechanism
Non-unitarity effects

ABSTRACT

The non-unitarity effects in leptonic flavor mixing are regarded as one of the generic features of the type-I seesaw model. Therefore, we explore these effects in the TeV-scale type-I seesaw model, and show that there exist non-trivial correlations among the non-unitarity parameters, stemming from the typical flavor structure of the low-scale seesaw model. In general, it follows from analytical discussions and numerical results that all the six non-unitarity parameters are related to three model parameters, while the widely studied parameters $\eta_{e\tau}$ and $\eta_{\mu\tau}$ cannot be phenomenologically significant simultaneously.

1. Introduction

During the past decade, experimental progress on neutrino masses and leptonic mixing has opened up a new window in searching for physics beyond the Standard Model (SM) of particle physics. Since neutrinos are massless particles in the SM, one usually extends the SM particle content in order to accommodate massive neutrinos. Among various theories of this kind, the seesaw mechanism [1] attracts a lot of attention in virtue of its naturalness and simplicity. In the conventional type-I seesaw model, three right-handed neutrinos are introduced and assigned large Majorana masses. In order to stabilize the masses of the light neutrinos at the sub-eV scale, the masses of right-handed neutrinos are usually chosen to be close to the Grand Unified (GUT) scale, i.e., 10^{16} GeV far above the electroweak scale $\Lambda_{\rm EW} \sim 100$ GeV. Thus, the typical type-I seesaw model suffers from lack of testability, since right-handed neutrinos are too heavy to be produced in current collider experiments. The Large Hadron Collider (LHC) will soon bring us a revolution in particle physics at the TeV-scale, which leads neutrino physics at the TeV-scale to an exciting direction. In this respect, the question, if we can find the answer of neutrino mass generation at the LHC, draws more and more attention.

In order for the type-I seesaw model to be testable, i.e., bringing the right-handed neutrino masses down to the TeV level, the small neutrino masses have to be effectively suppressed via other mechanisms rather than the GUT scale, such as radiative generation, small lepton number breaking, or neutrino masses from a higher than dimension-five effective operator (see, e.g. Ref. [2] and references therein). Besides, special cancellations in the contributions to the neutrino masses can also be employed to solve this problem. For instance, in the type-I+II seesaw model, one may assume the right-handed neutrino contributions to the light neutrino mass matrix to be comparable with the contributions originated from the triplet Higgs field, and the tiny left-handed neutrino masses are suppressed if there is a severe cancellation between these two mass terms. However, such a scheme seems implausible, since it involves strong fine-tuning between different and unrelated sources.

In this work, we will focus on a simple, but realistic, low-scale type-I seesaw model, in which a structural cancellation among the contributions from different right-handed neutrinos plays the key role of protecting neutrino masses [3]. Such a structural cancellation could naturally be regarded as the appearance of certain flavor symmetries stemming from some underlying dynamics. Furthermore, the mixing between heavy and light neutrinos results in observable non-unitarity (NU) effects in leptonic flavor mixing [4–6], which are usually parametrized by using so-called NU parameters. In what follows, we will discuss the possible NU effects in this framework in detail, and in particular, we will show that, due to the special flavor structure of the TeV-scale type-I seesaw, non-trivial correlations among the NU parameters exist, and not all the NU parameters can be phenomenologically significant in future neutrino oscillation experiments.

The remaining part of this work is organized as follows. In Section 2, we introduce the low-scale type-I seesaw model. Then, in Section 3, we explore in detail the typical features of the NU parameters in the model. A discussion on the possible effects in future neutrino oscillation experiments is also given. Section 4 is

* Corresponding author.
E-mail addresses: tommy@theophys.kth.se (T. Ohlsson), cgpopa@kth.se (C. Popa), zhanghe@kth.se (H. Zhang).

0370-2693
doi:10.1016/j.physletb.2010.07.053

devoted to numerical illustrations of the allowed parameter regions. Finally, a brief summary is presented in Section 5.

2. Low-scale type-I seesaw model

We first write out explicitly the Lagrangian responsible for neutrino Yukawa interactions and the Majorana mass term of right-handed neutrinos in the type-I seesaw model, viz.,

$$\mathcal{L} = -\overline{\ell_L}\tilde{\phi}Y_\nu^\dagger \nu_{\rm R} - \frac{1}{2}\overline{\nu_{\rm R}}M_{\rm R}\nu_{\rm R}^c + \text{h.c.}, \tag{1}$$

with $\tilde{\phi} = {\rm i}\tau_2\phi^*$, where $\ell_{\rm L}$, $\nu_{\rm R}$, and ϕ denote lepton doublets, right-handed neutrinos, and Higgs fields, respectively. Here Y_ν and $M_{\rm R}$ stand for the corresponding Yukawa couplings and right-handed neutrino mass matrix. If the right-handed neutrino scale is higher than the electroweak scale, one should integrate out the heavy degree of freedom of right-handed components in dealing with low-scale processes. Explicitly, at tree level, the only dimension-five operator is the Weinberg operator,

$$\mathcal{L}_W = \left(Y_\nu^T \frac{1}{M_{\rm R}} Y_\nu\right)_{\alpha\beta} \left(\overline{\ell_{{\rm L}\alpha}}\varepsilon\tau^i \ell_{{\rm L}\beta}^c\right)\left(\tilde{\phi}^T\varepsilon\tau^i\tilde{\phi}\right) + \text{h.c.} \tag{2}$$

After electroweak symmetry breaking, the SM Higgs field acquires a nonzero vacuum expectation value $v \simeq 174$ GeV, while the Majorana mass matrix of the light neutrinos then reads

$$m_\nu = -v^2 Y_\nu^T \frac{1}{M_{\rm R}} Y_\nu. \tag{3}$$

By defining $M_{\rm D} = vY_\nu$, Eq. (3) reproduces the ordinary type-I seesaw formula.

In the normal type-I seesaw model, the masses of the light neutrinos are purely suppressed by the ratio of the electroweak scale and the $B-L$ breaking scale, i.e., $v/M_{\rm R}$, and hence, the right-handed neutrino masses are usually chosen to be close to the GUT scale. In order to lower the $B-L$ scale to the scope of current colliders, i.e., the TeV-scale, one can simply assume the neutrino Yukawa couplings to be much smaller than those of other SM fermions. However, this will result in tiny couplings between the right-handed neutrinos and the charged leptons. Consequently, the production cross section of heavy neutrinos is also negligible. In this respect, one would like to maintain sizable Yukawa couplings and keep the neutrino masses stable simultaneously. In order to achieve this goal in the type-I seesaw framework, the contributions to neutrino masses from different right-handed neutrinos have to cancel, at least at leading order. The small neutrino masses can then be viewed as perturbations to the structure cancellation.

The necessary and sufficient conditions for such an exact cancellation require the following form of the Yukawa couplings [3]

$$Y_\nu = \begin{pmatrix} x_1 & ax_1 & bx_1 \\ x_2 & ax_2 & bx_2 \\ x_3 & ax_3 & bx_3 \end{pmatrix}, \tag{4}$$

where a and b are free parameters, and the relation between right-handed neutrino masses is given by

$$\frac{x_1^2}{M_1} + \frac{x_2^2}{M_2} + \frac{x_3^2}{M_3} = 0, \tag{5}$$

where we have already chosen a basis in which the right-handed neutrino mass matrix is diagonal. Using Eq. (3), one can easily prove that, under the above conditions, the neutrino masses vanish, namely, $m_\nu = 0$. Note that the radiative corrections induced by right-handed neutrinos, such as renormalization group running effects, may spoil the stability of small neutrino masses unless the right-handed neutrinos are nearly degenerate in mass as required by some flavor symmetric theories.

3. Non-unitarity of the leptonic mixing matrix

Besides the dimension-five operator discussed above, there exists a unique dimension-six operator [7]

$$\mathcal{L}_6 = C^6_{\alpha\beta}\left(\overline{\ell_{{\rm L}\alpha}}\tilde{\phi}\right){\rm i}\partial\!\!\!/\left(\tilde{\phi}^\dagger \ell_{{\rm L}\beta}\right), \tag{6}$$

where the coefficient is given by

$$C^6 = Y_\nu^\dagger \frac{1}{M_{\rm R}^\dagger}\frac{1}{M_{\rm R}} Y_\nu. \tag{7}$$

After the electroweak symmetry breaking, the dimension-six operator given in Eq. (6) leads to corrections to the kinetic energy terms for the light neutrinos. Therefore, in order to keep the neutrino kinetic energy canonically normalized, one has to rescale the neutrino fields by using the following transformation

$$\nu'_{{\rm L}\alpha} = \left(\delta_{\alpha\beta} + v^2 C^6_{\alpha\beta}\right)^{\frac{1}{2}}\nu_{{\rm L}\beta}. \tag{8}$$

Due to this field rescaling, the usual leptonic mixing matrix U, which relates the neutrino flavor basis and mass basis, is replaced by a non-unitary matrix as

$$N = \left(1 - \frac{v^2}{2}C^6\right)U = (1+\eta)U = RU, \tag{9}$$

where η is a Hermitian matrix containing totally nine parameters, i.e., six moduli and three phases governing the NU effects, and U diagonalizes the light neutrino mass matrix as $U^\dagger m_\nu U^* = {\rm diag}(m_1, m_2, m_3)$ with m_i being the masses of the light neutrinos.

Note that, different from the dimension-five operator, under the assumptions in the previous section, the dimension-six operator is not necessarily vanishing, since the flavor structure is different and C^6 is suppressed by the square of $M_{\rm R}$. Combining Eqs. (4)–(5) and (9), we can explicitly write down the NU parameters as

$$\eta = -\frac{v^2}{2}C^6 = \eta_0 \begin{pmatrix} 1 & a & b \\ a^* & |a|^2 & a^*b \\ b^* & ab^* & |b|^2 \end{pmatrix}, \tag{10}$$

where

$$\eta_0 = -\frac{v^2}{2}\left(\frac{|x_1|^2}{M_1^2} + \frac{|x_2|^2}{M_2^2} + \frac{|x_3|^2}{M_3^2}\right). \tag{11}$$

As a rough estimate, if we choose $M_i \sim$ TeV and the Yukawa couplings at order one, then $\eta \sim 0.1\%$ can be expected. In addition, the magnitudes of the NU parameters are constrained from universality test, rare lepton decays, and invisible width of Z-boson. The present bounds at 90% C.L. on the NU parameters are given by [4]

$$|\eta| < \begin{pmatrix} 2.0\times10^{-3} & 6.0\times10^{-5} & 1.6\times10^{-3} \\ \sim & 8.0\times10^{-4} & 1.1\times10^{-3} \\ \sim & \sim & 2.7\times10^{-3} \end{pmatrix}, \tag{12}$$

in which, the most severe constraint is that on the $e\mu$ element coming from the $\mu \to e\gamma$ decay.[1]

In the literature, these NU parameters are usually taken as free parameters. However, according to Eq. (10), the NU parameters are not independent in general. The correlations between these parameters stemming from some possible flavor symmetries can be viewed as a typical feature of the low-scale type-I seesaw model,

[1] Note that, in deriving these constraints, the condition $M_{\rm R} > \Lambda_{\rm EW}$ is assumed. In case of $M_{\rm R} < \Lambda_{\rm EW}$, the constraint on $\eta_{e\mu}$ is relaxed due to the restoration of the GIM mechanism.

and should be tested in the future neutrino oscillation experiments.

Concretely, it can be seen from Eq. (10) that the NU parameters are governed by the three independent parameters η_0, a, and b. The present restrictions on the elements of η are at percentage level, except a rather stringent bound $\eta_{e\mu} < 6.0 \times 10^{-5}$. Hence, to avoid severe unitarity constraints, one may expect either a or η_0 in Eq. (10) to be tiny. If a is very small, one may ignore the NU parameters proportional to a, and Eq. (10) can be simplified to

$$\eta \simeq \eta_0 \begin{pmatrix} 1 & 0 & b \\ 0 & 0 & 0 \\ b^* & 0 & |b|^2 \end{pmatrix}. \tag{13}$$

In this limit, both the $\eta_{e\mu}$ and $\eta_{\mu\tau}$ cannot be significant and $\eta_{e\tau}$ is the only possibly large NU parameter. On the other hand, in the case η_0 is very small while a and b are relatively large, the first row and column in η can be ignored, and one has approximately

$$\eta \simeq \eta_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & |a|^2 & a^* b \\ 0 & ab^* & |b|^2 \end{pmatrix}. \tag{14}$$

Therefore, $\eta_{e\tau}$ and $\eta_{\mu\tau}$ cannot be sizable simultaneously, although their current upper bounds are both within the sensitivity scope of a neutrino factory.

When neutrinos propagate in vacuum, in the ultra-relativistic limit $E \gg m_i$, the time evolution in the flavor basis is described by the Hamiltonian

$$H = \frac{1}{2E} \tilde{R}^* \big[U^* \cdot \mathrm{diag}\big(m_1^2, m_2^2, m_3^2\big) \cdot U^T \big] \big(\tilde{R}^*\big)^{-1}, \tag{15}$$

where the normalized $\tilde{R}_{\alpha\beta} \equiv R_{\alpha\beta} (RR^\dagger)_{\alpha\alpha}^{-\frac{1}{2}}$ is used instead of $R_{\alpha\beta}$ for the consistency between quantum states and fields. The transition amplitude from a neutrino flavor α to another neutrino flavor β after traveling a distance L can now be obtained as [8]

$$A_{\alpha\beta}(L) = \sum_i F^i_{\alpha\beta} \exp\left(-\mathrm{i} \frac{m_i^2 L}{2E} \right), \tag{16}$$

where $F^i_{\alpha\beta} = \sum_{\gamma,\rho} (\tilde{R}^*)_{\alpha\gamma} (\tilde{R})_{\beta\rho} U^*_{\gamma i} U_{\rho i}$. With the above definitions, the oscillation probability is then given by $P_{\alpha\beta}(L) \equiv |A_{\alpha\beta}(L)|^2$. A salient feature is that, in the case $\alpha \neq \beta$, $P_{\alpha\beta}(0)$ is not vanishing generally. Therefore, a flavor transition might already happen at the source even before the oscillation process, which is known as the zero-distance effect.

In general, a near detector at a short distance provides the best sensitivities to the NU parameters, since the standard oscillation effects in the unitary limit are suppressed with respect to the baseline length. In particular, in a future neutrino factory, a near detector with ν_τ detection is shown to be useful for studying NU effects [5]. In this respect, one may be interested in the flavor transitions in the appearance channels. For short enough distances, the oscillation amplitudes approximate to

$$A_{\alpha\beta}(L) \simeq A^{\mathrm{SM}}_{\alpha\beta}(L) + 2\eta^*_{\alpha\beta}, \tag{17}$$

where $A^{\mathrm{SM}}_{\alpha\beta}(L)$ denotes the oscillation amplitude of the unitary analysis. With respect to the NU parameters in the present model, i.e., Eqs. (13) and (14), the channels $\nu_e \to \nu_\tau$ and $\nu_\mu \to \nu_\tau$ [9] turn out to be the best options to search for the NU effects. On the other hand, if sizable NU effects are observed in both two channels, the simplest type-I seesaw model will be ruled out. The possible way out might then be theories possessing more than three heavy neutrinos, e.g. the inverse seesaw model [10] or theories with extra spatial dimensions [11].

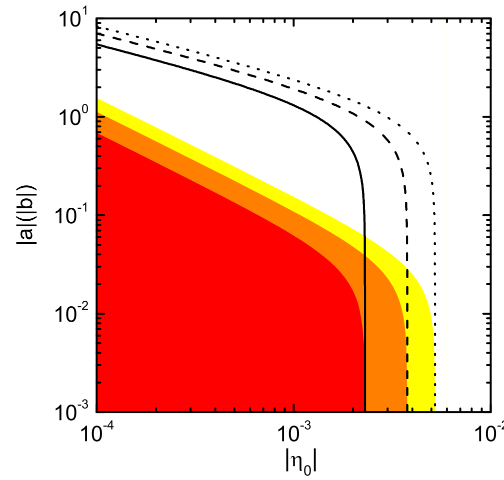

Fig. 1. Allowed regions of the model parameters a (colored regions), b (curves), and η_0 at 1σ (red, solid), 2σ (orange, dashed), and 3σ (yellow, dotted) C.L. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4. Numerical analysis

We proceed to perform a full scan of the parameter space of the model in order to obtain predictions for the NU parameters. For each set of these parameters, we compare the model predictions to the experimental data with a χ^2 function

$$\chi^2 = \sum_i \frac{(\rho_i - \rho_i^0)^2}{\sigma_i^2}, \tag{18}$$

where ρ_i^0 are assumed to be zero for the central values of the NU parameters, σ_i the corresponding 1σ absolute error, and ρ_i the predicted values of η's. In our numerical analysis, we make use of the current bounds given in Eq. (12). Note that, since neutrino masses are assumed to be generated via other mechanisms (e.g. deviations from the exact structure cancellations), we do not consider experimental constraints on neutrino masses and leptonic flavor mixing parameters in our analysis. Discussions on the neutrino mass generation in the current framework can be found in Refs. [3].

In Fig. 1, we show the allowed parameter space of a, b, and η_0 at 1σ, 2σ, and 3σ C.L. One can observe from the plot that there exists an upper bound on η_0, which results from the relation $\eta_0 = \eta_{ee}$ according to Eq. (10). Furthermore, for a fixed η_0, the allowed regions of b are larger than those of a, i.e., the upper bound of b is about one order of magnitude larger than that of a. This is in agreement with our analytical analysis, since $\eta_{e\mu}$ sets only a strong constraint on a but not on b. For a very tiny η_0, a and b can be arbitrarily chosen without suffering from stringent unitarity constraints, i.e., their upper bounds approach infinity. However, as we mentioned before, for a realistic type-I seesaw model with sub-eV scale neutrino masses, these parameters cannot be arbitrarily small unless another mechanism responsible for the masses of the light neutrinos is considered.

According to Eq. (17), the phenomenologically interesting NU parameters are the off-diagonal elements in η. Since $\eta_{e\mu}$ is strongly constrained experimentally, the remaining ones are $\eta_{e\tau}$ and $\eta_{\mu\tau}$, whose allowed regions[2] are illustrated in Fig. 2. One can observe that both $\eta_{e\tau}$ and $\eta_{\mu\tau}$ can reach their upper bounds

[2] Given the fact that there is no experimental information on the leptonic CP-violation until now, we do not include the CP-violating phases of η's, and only show the constraints on the absolute values of these NU parameters.

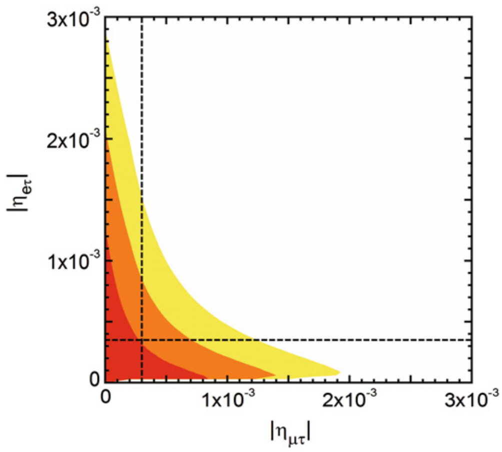

Fig. 2. Allowed regions of the NU parameters $|\eta_{e\tau}|$ and $|\eta_{\mu\tau}|$ at 1σ (red), 2σ (orange), and 3σ (yellow) C.L. The dashed lines in the figure correspond to the sensitivities to these NU parameters (at 90% C.L.) in a near OPERA-like tau-detector of a neutrino factory (parent muon energy $E = 25$ GeV). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

in general. However, as we expected from the above analysis, in order for one of them to be sizable, the other one has to be suppressed, reflecting the underlying correlations among the NU parameters. For example, in the case of $\eta_{e\tau} > 10^{-3}$, a severe bound $\eta_{\mu\tau} \lesssim 3 \times 10^{-4}$ (at 2σ C.L.) can be read off from the plot. Similarly, in the case of $\eta_{\mu\tau} > 10^{-3}$, one has $\eta_{e\tau} \lesssim 3 \times 10^{-4}$ (at 2σ C.L.). For comparison, in the plot we also show the discovery potential of the NU parameters by using an OPERA-like near tau-detector of a future neutrino factory (see detailed discussions on the detector setup in e.g. Refs. [5]). The regions on the right-hand side of the vertical line and above the horizontal line can be well searched for with such an experimental setup. Therefore, besides the search of the NU effects, it can also shed some light on distinguishing the possible new physics behind the NU effects.

5. Summary and conclusion

In this Letter, we have studied the NU effects from the low-scale type-I seesaw model. We have pointed out that in the realistic low-scale type-I seesaw model without unacceptable lepton number violations, there exists a non-trivial flavor structure of the NU parameters, which originates from the structural cancellations of the neutrino Yukawa couplings. The underlying correlations among the NU parameters have been established, and the allowed parameter spaces have been illustrated. In view of the current constraints on the NU parameters, we have found that, in the low-scale type-I seesaw model, there exists only one phenomenologically interesting NU parameter, i.e., either $\eta_{e\tau}$ or $\eta_{\mu\tau}$. This flavor structure can be viewed as a distinctive feature of the low-scale type-I seesaw model, and can be tested in the future experiments, in particular, in the near tau-detector of a neutrino factory. In addition to the neutrino oscillation experiments, direct searches of right-handed neutrinos at colliders also rely on the NU parameters. Especially, the signals of tri-lepton final states with transverse missing energy at the LHC provide us with the capability of revealing the underlying nature of right-handed neutrinos. In conclusion, the low-scale type-I seesaw features a very distinctive flavor structure, and a combined analysis of both collider signatures and neutrino oscillation experiments will be very useful to obtain knowledge on the physics behind the right-handed neutrinos.

Acknowledgements

This work was supported by the Swedish Research Council (Vetenskapsrådet), contract No. 621-2008-4210 (T.O.) and the Royal Institute of Technology (KTH), project No. SII-56510 (H.Z.). H. Zhang acknowledges Xiangdong Ji for the hospitality at Peking University and Shanghai Jiaotong University where part of this work was performed.

References

[1] P. Minkowski, Phys. Lett. B 67 (1977) 421;
T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proc. Workshop on the Baryon Number of the Universe and Unified Theories, 1979, p. 95;
M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, D. Freedman (Eds.), Supergravity, 1979, p. 315;
R.N. Mohapatra, G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.
[2] F. Bonnet, D. Hernández, T. Ota, W. Winter, JHEP 0910 (2009) 076, arXiv:0907.3143.
[3] A. Pilaftsis, Z. Phys. C 55 (1992) 275, hep-ph/9901206;
Kersten J., A.Y. Smirnov, Phys. Rev. D 76 (2007) 073005, arXiv:0705.3221;
H. Zhang, S. Zhou, Phys. Lett. B 685 (2010) 297, arXiv:0912.2661;
R. Adhikari, A. Raychaudhuri, 2010; arXiv:1004.5111.
[4] S. Antusch, C. Biggio, E. Fernández-Martínez, M.B. Gavela, J. López-Pavón, JHEP 0610 (2006) 084, hep-ph/0607020;
S. Antusch, J.P. Baumann, E. Fernández-Martínez, Nucl. Phys. B 810 (2009) 369, arXiv:0807.1003.
[5] J. Tang, W. Winter, Phys. Rev. D 80 (2009) 053001, arXiv:0903.3039;
D. Meloni, T. Ohlsson, W. Winter, H. Zhang, JHEP 1004 (2010) 041, arXiv:0912.2735.
[6] E. Fernández-Martínez, M.B. Gavela, J. López-Pavón, O. Yasuda, Phys. Lett. B 649 (2007) 427, hep-ph/0703098;
Z.-z. Xing, Phys. Lett. B 660 (2008) 515, arXiv:0709.2220;
T. Ota, S. Goswami, Phys. Rev. D 78 (2008) 033012, arXiv:0802.1434;
Z.-z. Xing, S. Zhou, Phys. Lett. B 666 (2008) 166, arXiv:0804.3512;
S. Luo, Phys. Rev. D 78 (2008) 016006, arXiv:0804.4897;
G. Altarelli, D. Meloni, Nucl. Phys. B 809 (2009) 158, arXiv:0809.1041;
M. Malinský, T. Ohlsson, H. Zhang, Phys. Rev. D 79 (2009) 073009, arXiv:0903.1961;
M. Blennow, E. Fernández-Martínez, Comput. Phys. Comm. 181 (2010) 227, arXiv:0903.3985;
S. Antusch, M. Blennow, E. Fernández-Martínez, J. López-Pavón, Phys. Rev. D 80 (2009) 033002, arXiv:0903.3986;
W. Rodejohann, Europhys. Lett. 88 (2009) 51001, arXiv:0903.4590;
M. Malinský, T. Ohlsson, Z.-z. Xing, H. Zhang, Phys. Lett. B 679 (2009) 242, arXiv:0905.2889;
X.-G. He, S. Oh, J. Tandean, C.-C. Wen, Phys. Rev. D 80 (2009) 073012, arXiv:0907.1607;
Z.-z. Xing, Phys. Lett. B 679 (2009) 255, arXiv:0907.3014;
W. Chao, S. Luo, Z.-z. Xing, S. Zhou, Phys. Rev. D 77 (2008) 016001, arXiv:0709.1069;
P.S.B. Dev, R.N. Mohapatra, Phys. Rev. D 81 (2010) 013001, arXiv:0910.3924;
S. Antusch, S. Blanchet, M. Blennow, E. Fernández-Martínez, JHEP 1001 (2010) 017, arXiv:0910.5957;
W. Rodejohann, Phys. Lett. B 684 (2010) 40, arXiv:0912.3388.
[7] A. Broncano, M.B. Gavela, E.E. Jenkins, Phys. Lett. B 552 (2003) 177, hep-ph/0210271;
A. Broncano, M.B. Gavela, E.E. Jenkins, Nucl. Phys. B 672 (2003) 163, hep-ph/0307058;
A. Abada, C. Biggio, F. Bonnet, M.B. Gavela, T. Hambye, JHEP 0712 (2007) 061, arXiv:0707.4058;
M.B. Gavela, T. Hambye, D. Hernández, P. Hernández, JHEP 0909 (2009) 038, arXiv:0906.1461.
[8] T. Ohlsson, H. Zhang, Phys. Lett. B 671 (2009) 99, arXiv:0809.4835;
D. Meloni, T. Ohlsson, H. Zhang, JHEP 0904 (2009) 033, arXiv:0901.1784.
[9] A. Donini, D. Meloni, P. Migliozzi, Nucl. Phys. B 646 (2002) 321, hep-ph/0206034;
A. Donini, K.-i. Fuki, J. López-Pavón, D. Meloni, O. Yasuda, JHEP 0908 (2009) 041, arXiv:0812.3703.
[10] R.N. Mohapatra, J.W.F. Valle, Phys. Rev. D 34 (1986) 1642.
[11] S. Bhattacharya, P. Dey, B. Mukhopadhyaya, Phys. Rev. D 80 (2009) 075013, arXiv:0907.0099;
M. Blennow, H. Melbéus, T. Ohlsson, H. Zhang, 2010; arXiv:1003.0669.